Problem 1

(A∩B)×(A∪B) = ∅, 若(A∩B) ≠ ∅ 且(A∪B) ≠ ∅, ∃x∈A, ∃y∈B,

则∃(x, y)∈(A∩B)×(A∪B), 矛盾, 可知((A∩B) = ∅)∨((A∪B) = ∅).

假设(A∩B) ≠ ∅, 根据析取三段论, (A∪B) = ∅.

A = ∅, B = ∅, (A∩B) = ∅, 矛盾, 故(A∩B) = ∅.

Problem 2

a) {x∈Z | x≥1} b) ∅ c) {x∈Z | x<0∧x>1} = {x∈Z | x≠0∧x≠1}

Problem 3

a) 不能, 取A={0}, B={1}, C=U, A∪C = U, B∪C = U, A∪C = B∪C, A≠B.

b) 不能, 取A={0}, B={1}, C=∅, A∩C = ∅, B∩C = ∅, A∩C = B∩C, A≠B.

c) 能. 取x∈A, x∈A∪C, 又A∪C = B∪C, 则x∈B∪C.

1° 若x∈C, 得x∈A∩C, 又A∩C = B∩C, 则x∈B∩C, x∈B.

2° 若x∉C, 由x∈B∪C得x∈B.

综上所述可知A⊆B, 同理取y∈B可证B⊆A, A = B.

Problem 4

A⊆B可表示为∀x(x∈A→x∈B).

∀x(x∈A→x∈B)≡∀x(x∉A∨x∈B)≡∀x(x∉~B∨x∈~A)≡∀x(x∈~B→x∈~A)

由∀x(x∈~B→x∈~A)可得~B⊆~A.

Problem 5

a) A⊕A = (A∪A)-(A∩A) = A-A = A∩~A = ∅

b) A⊕U = (A∪U)-(A∩U) = U-A = U∩~A = ~A

Problem 6

对Ai = {x | x∈Z∧x≤i}, Ai ⊆ Ai+1, 则:

a) ∪n i=1 Ai = A1∪A2∪…∪Ai = Ai = {x | x∈Z∧x≤i}

b) ∩n i=1 Ai = A1∩A2∩…∩Ai = A1 = {x | x∈Z∧x≤1}

Problem 7

ρ(A) = { ∅, {∅}, {{1}}, {{1}, ∅}, {{1, 2}}, {{1, 2}, ∅}, {{1, 2}, {1}}, {{1, 2}, {1}, ∅} }

a) ∪ρ(A) = ∅∪{∅}∪{{1}}∪{{1}, ∅}∪{{1, 2}}∪{{1, 2}, ∅}∪{{1, 2}, {1}}∪{{1, 2}, {1}, ∅}

∪ρ(A) = {{1, 2}, {1}, ∅} = A

b) ∩∪ρ(A) = ∩A = {1, 2}∩{1}∩∅ = ∅